Available online at www.sciencedirect.com
SCIENCE DIRECT®
International Journal of Heat and Mass Transfer 47 (2004) 3577–3581
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

Transient laminar conjugate heat transfer of a rotating disk: theory and numerical simulations

T. Indinger [a,*], I.V. Shevchuk [b,1]

[a] Institut für Strömungsmechanik, Technische Universität Dresden 01062, Dresden, Germany
[b] Institute of Engineering Thermophysics, Ukrainian National Academy of Sciences, 2a, Zhelyabov St., Kiev 03057, Ukraine

Received 5 June 2003; received in revised form 23 December 2003
Available online 8 April 2004

Abstract

The study considers transient laminar heat transfer of a rotating disk heated to a constant initial temperature and sudden subjection to unsteady cooling by still air. Both numerical simulations of conjugate heat transfer inside the disk and convection between the disk and air, as well as a self-similar solution show that the heat transfer coefficient becomes time independent very quickly and equal to its value at steady-state conditions. It appeared that the widely employed solution of unsteady one-dimensional heat conduction in a semi-infinite plate overestimates significantly the disk temperature and consequently the heat transfer coefficient calculated from this solution using known experimental instant disk surface temperature.

Keywords: Conjugate; Heat transfer; Rotating

1. Introduction

Rotating-disk systems are widely known in turbomachinery, computer-disk drives, rotating dust separators etc. Unsteady heat transfer of rotating disks is frequently used in transient experimental techniques. Such techniques currently employing liquid crystals are based on the commonly known fact that after a certain period of time since the beginning of the cooling process, the surface heat transfer coefficient becomes a time-independent function equal to its value of steady-state heat transfer of the same body under analogous boundary conditions. The heat fluxes in these cases can easily be calculated from rather simple analytical solutions for unsteady heat conduction inside solid bodies at known surface temperatures.

This problem actual for the present study was stated in [1]. A Plexiglas® rotating disk ($b = 0.123$ m, $s = 0.01$ m) was placed in a thermally insulated box and heated up to a constant temperature of $T_{w,i} = 40$ °C (Fig. 1). Then the box cover was suddenly removed and the disk started cooling down to room temperature of $T_\infty = 24$ °C without additional input of heat. Unsteady surface temperatures were measured by means of liquid crystals applied onto the disk surface as ring-like bands. The heat transfer coefficient was computed from the solution of the inverse one-dimensional transient problem of heat conduction inside the disk being assumed to be semi-infinite in z-direction

$$F_t(t) = \frac{T_w(t) - T_\infty}{T_{w,i} - T_\infty} = \exp(\gamma^2) \cdot \operatorname{erfc}(\gamma) \quad \text{with} \quad \operatorname{erfc}(\gamma) = 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k \gamma^{2k+1}}{k!(2k+1)}. \tag{1}$$

* Corresponding author. Tel.: +49-351-463-34897; fax: +49-351-463-35246.
E-mail address: thomas.indinger@ism.mw.tu-dresden.de (T. Indinger).
[1] Present address: Currently Alexander von Humboldt Research Fellow, Institut für Luft-und Raumfahrttechnik, Technische Universität Dresden, 01062 Dresden, Germany

0017-9310/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2004.01.023

Nomenclature

a	thermal diffusivity
b	outer radius of disk
Bi	Biot number $0.5hs/k_w$
$F_t(t)$	relative instant wall temperature, $(T_w(t) - T_\infty)/(T_{w,i} - T_\infty)$
Re_φ	rotational Reynolds number $\omega b^2 v$
s	thickness of the disk
T	temperature
t	time
$x = r/b$	non-dimensional radial coordinate
Fo	Fourier number $4a_w t/s^2$
h	heat transfer coefficient
k	thermal conductivity
K_1	constant in Eq. (2)
Nu_b	Nusselt number $q_w b/[k(T_w - T_\infty)]$
Pr	Prandtl number, v/a
q_w	local heat flux at the wall
r, φ, z	radial, tangential and axial coordinate

Greek symbols

v	kinematic viscosity
ρ	density
ω	angular speed of rotation of the disk

Subscripts

i	initial conditions ($t = 0$)
w	wall ($z = 0$)
∞	Infinity

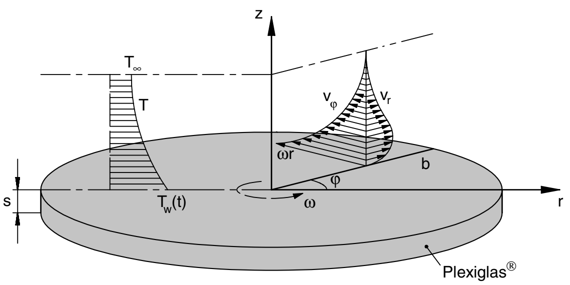

Fig. 1. Geometrical arrangement of a rotating disk in still air.

Here $T_w(t)$ is the instant disk surface temperature, $\gamma = h\sqrt{a_w t}/k_w$. Key assumption thereby is the heat transfer coefficient h being independent of time [1,2].

Authors [3] have shown that resulting radial distributions of h for laminar heat transfer in experiments [1] are in significant disagreement with theoretical solutions [4–7] and also with reliable experimental data [5,8,9]. This heat transfer characteristically value can be described by the following equation

$$Nu_b = K_1 Re_\varphi^{1/2}, \tag{2}$$

where $K_1 = 0.326$ at $Pr = 0.71$ and $T_w = \text{const}$. Distributions of h found in [1] decreased radially from the axis of rotation to the disk rim by two times, while all known correlations applicably to the experimental conditions [1] predict constant values of h, which agrees with experiments [1] only near the disk's rim. Unfortunately, authors [1] did not present experimental data for $F_t(t)$.

Studies undertaken in [10,11] showed that the semi-infinite wall assumption becomes invalid once the non-dimensional time value at which the measurements were taken exceeds a certain threshold value. This value expressed in terms of the Fourier number is equal to $Fo = 1/4$ in accordance with the classical theory [10] or $Fo = 1$ according to [11]. The reason for this restriction is that at $Fo > 1/4$ [10] (or 1 [11]) the temperature of the whole disk diverts from the initial value $T_{w,i}$ and thus the entire disk takes part in the heat transfer process.

The objective of this research is the numeric modeling of the conjugate two-dimensional transient problem of laminar heat transfer in air and rotating disk. The solution to be obtained should give an answer to the question whether the transient experimental technique used in [1,2] and elsewhere can provide reliable and adequate data for heat transfer coefficients of rotating disks including more complicated conditions.

2. Analytical solution for a disk of finite thickness

Concerning a disk of a finite thickness s (Fig. 1) with the same heat transfer coefficients at $z = 0$ and $z = -s$, the function $F_t(t)$ is [12]

$$F_t(t) = \sum_{i=1}^{\infty} \frac{2\sin(\mu_n)\cos(\mu_n)}{\mu_n + \sin(\mu_n)\cos(\mu_n)} \exp(-\mu_n^2 Fo), \tag{3}$$

$$\cot(\mu_n) = \mu_n/Bi, \tag{4}$$

where eigenvalues μ_n are defined by Eq. (4). The physical properties of Plexiglas® are [1] $k_w = 0.19$ W/(m^2 K), $a_w = 1.086 \times 10^{-7}$ m^2/s and of air [12] $k = 0.02624$ W/(m^2 K), $a = 2.216 \times 10^{-5}$ m^2/s, $Pr = 0.71$. It follows for the investigated problem $Bi = 0.395$. The Fourier number value at $t = 69$ s is $Fo = 0.3$, and with an inaccuracy not exceeding 0.37%, function $F_t(t)$ can be calculated from Eq. (3) using only the first term of the Fourier series [12].

According to Eq. (3), function $F_t(t)$ is significantly lower (by 2–4 times at $t > 700$ s) than function $F_t(t)$

given by Eq. (1), while K_1 is the same for both functions $F_t(t)$. Consequently, if the real cooling rate in experiments for the instant moment of time t exceeds twice the cooling rate predicted by Eq. (1), the use of Eq. (1) to restore the experimental value of the heat transfer coefficient will lead to obtaining the data for h exceeding twice its real values. This conclusion is validated below via numerical simulations using the CFD code CFX-5.

3. Numerical simulations

3.1. Computational domain and mesh

The commercially available program package CFX-5 by ANSYS Inc. was employed for the numerical simulations, whereas a case-sensitive mesh generated manually by the Patran volume mesher was used instead of that provided by the standard automatic mesher implemented in CFX-5. Because of the axisymmetric character of the problem, the overall computational domain comprised only a sector of 45° angle with a periodic boundary condition specified on each side. This domain consisted of two parts: a stationary fluid domain with the radius and height of 0.5 m and a rotating fluid domain that contained a solid sub-domain, namely the disk itself (heated by an energy source) in accordance with the experimental setup of [1]. The rotating and stationary domains are connected to each other by a so-called frozen rotor domain interface. The outer and top boundaries were defined as walls with a free slip condition and the fixed (ambient) temperature of $T_\infty = 297.15$ K. All lower surfaces of the computational domain were symmetry planes. These preconditions allowed the simulation of conjugate heat transfer of a freely rotating heated disk with finite dimensions in a stationary fluid domain where the outer boundaries did not affect fluid flow and heat transfer intensity in the near-wall region.

Meshing this domain, mainly hexahedrons and a few prisms with 18 elements in angular direction were used. The overall 333,520 nodes were partitioned in such a way that 11% were located inside the disk zone, 54% were used to model the rotating fluid domain with an extension of 10 mm around the solid disk and the remaining 35% were necessary to realize the stationary region. Preliminary investigations of the mesh structure and necessary density in the rotating domain showed that the manually created structured mesh was necessary for obtaining really smooth distributions of heat transfer values in the disk surface area according to self-similar solution [4–6]. In order to reduce the overall number of nodes, an unstructured coarser mesh was used in the outer stationary part of the whole computational domain. This was sufficient to simulate the ambience of still air.

3.2. Validations of steady-state fluid flow and heat transfer

In order to validate the model with its boundary conditions described in the previous section, several runs of simulations at $\omega = 52.36$ 1/s (or 500 rpm) were performed to get a well converging solution for fluid flow and heat transfer parameters. Target (normalized log) residual for all items was set to 1×10^{-6}. To reach this value, a timestep of $10 \times 1/\omega$ has been used during the start-up of the simulation, which finally needed to be reduced to $0.1 \times 1/\omega$.

In the area around the domain center ($x = 0.3$) an undisturbed fluid flow occurred without any influence of flow around the disk's rim, whereas such a disturbed flow took place at $x \geqslant 0.6$. Numerical simulations of the velocity and temperature profiles agreed well with the self-similar solutions [4–7]. The average value of $K_1 = 0.341$ (at $x \leqslant 0.6$) is in close consistence with the self-similar value $K_1 = 0.326$ for $Pr = 0.71$ and $T_w = \text{const}$. Data for the conduction heat transfer inside the disk was in accordance with the analytical solution [12]. This steady-state solution proved suitable for providing initial velocity, pressure and temperature distributions for transient simulations.

3.3. Results for transient heat transfer

The temperature, velocity and pressure fields obtained from the steady-state simulations described above were used as initial conditions for the transient run. Switching off the energy source (used to heat the disk during the steady-state run) allowed analyzing unsteady laminar heat transfer of a rotating disk in still air at the same angular speed of the rotating domain $\omega = 52.36$ 1/s. The calculation was carried out during 1500 s of physical time. Within this period the average disk surface temperature decreased from its initial value $T_w = 312.60$ K to $T_w = 298.80$ K. The surrounding air temperature was kept constant at $T_\infty = 297.15$ K.

The non-dimensional variable $F_t(t)$ reached the value of 0.106 when computations were terminated. Variation of this quantity versus time is shown in Fig. 2 and compared with Eqs. (1) and (3), respectively. The function $F_t(t)$ computed using CFX-5 is in good agreement with Eq. (3). Thus, the real cooling rate of the relatively thin ($s = 0.01$ m) Plexiglas® disk exceeds very significantly indeed the cooling rate predicted by Eq. (1). The reason for this discrepancy is that Eq. (1) is valid for high Biot numbers that refer to a disk of semi-infinite thickness or made of a material with low thermal conductivity k_w. Such a disk should indeed cool down much more slowly than a thin Plexiglas® disk used in experiments [1]. (It is easy to see that Eqs. (1) and (3) can provide the same numerical results at $Bi \to \infty$).

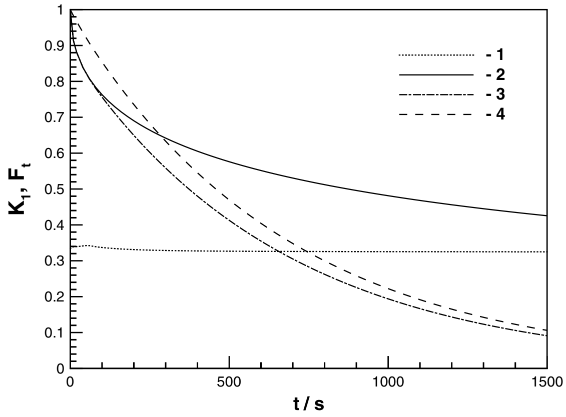

Fig. 2. Variation of K_1 and F_t versus time according to self-similar solution and numerical simulations using CFX-5 code. 1—K_1; 2—$F_t(t)$, Eq. (1); 3—$F_t(t)$, Eq. (3); 4—$F_t(t)$, CFX-5.

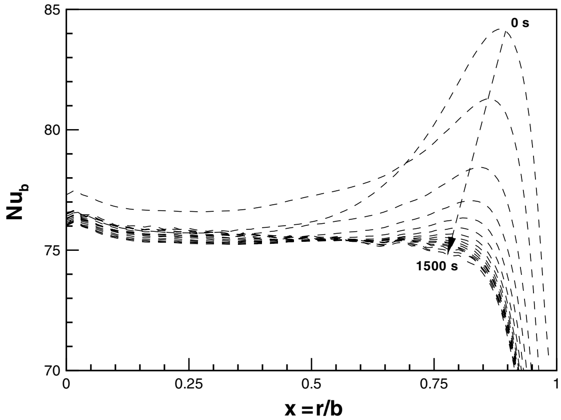

Fig. 3. Transitional behavior of the Nusselt number out of CFX-5 simulations from $t = 0$ to $t = 1500$ s. Dashed lines are representing intermediate distributions, taken every 100 s.

Thus, wrong choice of the Eq. (1) to process the transient experimental data in [1] is most probably the reason for significant deviation of the results [1] for the Nusselt number from Eq. (2) at $K_1 = 0.326$. Therefore, Eq. (3) should be used in transient experimental techniques instead of Eq. (1) to restore experimental values of heat transfer coefficients. As mentioned above, at $Fo = 0.3$ function $F_t(t)$ can be computed from Eq. (3) using only the first term of the Fourier series (with an inaccuracy less than 0.37%), which substantially simplifies the task of an experimentalist.

The high quality of simulations can be also confirmed by the excellent agreement of the constant K_1 with the self-similar solution also presented in Fig. 2. The disk cooled down very homogeneously. The temperature was nearly constant up to $x = 0.5$ at each point of time and decreased subsequently to the value at the disk's edge. The decreased value of T_w at $x = 1$ is of course the result of cooling down the outer cylindrical surface of the disk. The profile of T_w flattened steadily until the ambient temperature T_∞ was almost reached at $t = 1500$ s.

The main point of interest is the behavior of the Nusselt number. Radial distributions of Nu_b are presented in Fig. 3 for every 100 s starting at the beginning of the cooling process. The influence of the flow near the axis of rotation can be seen up to approximately $x = 0.2$. This resulted in a time-independent moderate bump in the profile of Nu_b. At each time step the Nusselt number is approximately constant between $x = 0.2$ and $x = 0.6$.

The significant peak of Nu_b at the outer part of the disk surface in the initial distribution at $t = 0$ s gradually vanishing with time is caused by the non-uniform initial distribution of the disk surface temperature $T_{w,i}$. The noticeably decreasing radial profile of $T_{w,i}$ causes radial heat conduction from the region of $x \leqslant 0.6$ to the outer part of the disk. After a while, surface temperature becomes more uniform, while a certain part of the heat accumulated by the disk is transferred to the ambient air by the outer cylindrical surface of the disk. These two effects eventually depress the peak of the Nusselt number at $x \approx 0.8$ mentioned afore.

Disregarding the effects near rotation axis and outer radius of the disk, an average value of the Nusselt number for the undisturbed region $0.2 \leqslant x \leqslant 0.6$ was calculated and plotted versus time. As evident in Fig. 3, the Nusselt number becomes practically time independent after about 300 s of physical time. After this period of time, the standard deviation from the mean value of $Nu_b = 75.497$ is only 0.103. This mean value of Nu_b is in excellent agreement with Eq. (2), which is based on the self-similar solution and reliable experiments [5,8,9] and others.

4. Conclusions

Using CFD code CFX-5, numerical simulations of the problem of unsteady conjugate laminar heat transfer of a rotating disk (at steady-state fluid flow conditions) showed that the heat transfer coefficient very quickly becomes time independent and equal to its value of steady-state conditions, confirmed by known reliable experiments [5,8,9] and others. It was shown that the solution for unsteady one-dimensional heat conduction in a semi-infinite plate employed in [1,2] overestimates significantly the disk surface temperature and consequently the heat transfer coefficient calculated from this solution using known experimental instant disk surface temperature. The alternative method may be the use of the solution for unsteady one-dimensional heat conduction in a finite-thickness plate given by Eq. (3), which degenerates at $Fo \geqslant 0.3$ just to the first term of the Fourier series.

Acknowledgements

The authors gratefully acknowledge the support provided by the Research Fellowships of Humboldt Foundation and German Academic Exchange Service (DAAD) for realizing research stays of I.V. Shevchuk at Dresden University of Technology. The authors also acknowledge ANSYS CFX, AEA Technology GmbH, Otterfing, Germany for using CFX-5 software and the support provided during this research.

References

[1] N. Saniei, X.T. Yan, An experimental study of heat transfer from a disk rotating in an infinite environment including heat transfer enhancement by jet impingement cooling, J. Enhanc. Heat Transfer 7 (2000) 231–245.

[2] G.D. Lock, F. Daguze, B.J. Syson, J.M. Owen, The application of thermal imaging techniques to determine heat transfer coefficients for slow thermal transients, In: Proceedings of the Thirteenth Symposium on Measuring Techniques for Transonic and Supersonic Flows in Cascades and Turbomachines, Zürich, September 5-6, 1996, pp. 1–7.

[3] I.V. Shevchuk, N. Saniei, X.T. Yan, Effect of the wall temperature distribution in direct and inverse problems of laminar heat transfer over a free rotating disk. In: Proceedings of ASME NHTC'01: 35th National Heat Transfer Conference, Anaheim, California, USA, June 10–12, 2001, Paper NHTC2001-20196, pp. 1–8.

[4] L.A. Dorfman, Hydrodynamic resistance and the heat loss of rotating solids, Olyver and Boyd, Edinburgh, 1963, pp. 9–118.

[5] J.M. Owen, R.H. Rogers, Flow and heat transfer in rotating-disk systems, in: Rotor-Stator-Systems, vol. 1, Research Studies Press Ltd., Taunton, England, 1989, pp. 93–124.

[6] I.V. Shevchuk, Effect of the wall temperature on laminar heat transfer in a rotating disk: an approximate analytical solution, High Temp. 39 (4) (2001) 637–640.

[7] I.V. Shevchuk, An improved enthalpy-thickness model for predicting heat transfer of a free rotating disk using an integral method, In: Proceedings of ASME NHTC'01: 35th National Heat Transfer Conference, Anaheim, California, USA, June 10–12, 2001, Paper NHTC2001-20195, pp. 1–8.

[8] C.J. Elkins, J.K. Eaton, 1997. Heat transfer in the rotating disk boundary layer, Stanford University, Department of Mechanical Engineering, Thermosciences Division Report TSD-103, pp. 1–206.

[9] G. Cardone, T. Astarita, G.M. Carlomagno, Infrared heat transfer measurements on a rotating disk, Opt. Diagn. Eng. 1 (2) (1996) 1–7.

[10] D.L. Schultz, T.V. Jones, Heat transfer measurements in short duration hypersonic facilities, Aeronautical RD AGARDOGRAPH (NATO Advisory Group) vol. 165 (1973).

[11] G. Vogel, B. Weigand, A new evaluation method for transient liquid crystal experiments. In: Proceedings of ASME NHTC'01: 35th National Heat Transfer Conference, Anaheim, California, USA, June 10–12, 2001, Paper NHTC2001–20250, 1–6.

[12] A.V. Luikov, Analytical Heat Diffusion Theory, Academic Press, New York, 1968, pp. 191–216.